2. Gravitation

Kepler's for celestial bodies

Kepler's First Law (Law of Orbits)

- Every planet revolves around the sun in an elliptical orbit. The sun is situated at one foci of the ellipse.

Kepler's Second Law (Law of Areas)

- The line joining a planet to the sun sweeps out equal areas in equal intervals of time, i.e., the areal velocity of the planet around the sun is constant

Kepler's Third Law (Law of Periods)

- The square of the time period of revolution of a planet around the sun is directly proportional to the cube of the semi major axis of its elliptical orbit.

- **Universal law of gravitation**

$$|\bar{F}| = G\frac{m_1 m_2}{r^2}$$

$$\bar{F} = G\frac{m_1 m_2}{r^2}(-\hat{r}) = -G\frac{m_1 m_2}{r^2}(\hat{r})$$

- **For a point mass**

  The point of attraction between a hollow spherical shell of uniform density and a point mass situated outside is just as if the entire mass of the cell is concentrated at the center of the shell.

  The force of attraction due to a hollow spherical shell of uniform density, on a point mass inside it is zero.

- **Gravitation Constant, G** = 6.67×10^{-11} Nm^2/kg^2

- **Earth's gravitational acceleration**,

$$g = \frac{F}{m} = \frac{GM_E}{R_E^2}$$

- **Below/above Earth's surface**

$$g(h) = g\left(1 - \frac{2h}{R_E}\right) \quad [h = \text{ Height from Earth's surface}]$$

$$g(d) = g\left(1 - \frac{d}{R_E}\right) \quad [d = \text{ Depth from Earth's surface}]$$

- **Gravitational potential energy**

$$W_{12} = \int_{r_1}^{r_2} \frac{GMm}{r^2} dr$$

$$W(r) = -\frac{GM_E m}{r} + W_1 \quad (\text{Valid for } r > R_E)$$

- For any two bodies, $V = -\frac{Gm_1 m_2}{r}$

- **Escape velocity**

$$(V_i)_{min} = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{2gR_E} \approx 11.2 \text{ km/s}$$

This is the escape velocity for earth.

- Total energy of a mass m moving with a velocity v in the vicinity of another mass M,

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

- For circular path of radius a, $E = \frac{GMm}{2a}$

- **Earth's satellite**
  - Speed and time period

$$\text{Speed, } V = \sqrt{\frac{GM_E}{(R_E + h)}}$$

$$\text{Time period, } T^2 = k(R_E + h)^3 \quad \left(k = \frac{4\pi^2}{GM_E}\right) \quad [\text{Kepler's } 3^{rd} \text{ law}]$$

- For satellite very close to earth, $T = 2\pi\sqrt{\frac{R_E}{g}}$

- **Energy of orbiting satellite**

$$P.E. = -\frac{GmM_E}{(R_E + h)}$$

$$K.E. = \frac{GmM_E}{2(R_E + h)}$$

$$\text{Total Energy} = -\frac{GmM_E}{2(R_E + h)}$$

- **Geostationary satellite**
  - Height is about 36000 km.
  - $T = 24$ h
  - Used in television and other communication and broadcasting (INSAT group)
- **Polar satellites** are low-altitude satellites. They orbit the earth many times during a day. They are used in remote sensing, meteorology, and environment studies.
- **Weightlessness**
  - In space weightlessness is experienced because gravitational pull is used up in providing centripetal force.
  - In free fall weightlessness is experienced because there is no normal force acting opposite to the force of gravity.
- **Parking orbit** is a temporary orbit around the Earth where the satellite is temporarily parked before it is launched to its desired orbit.